

Next-to-leading order QCD corrections to W^+W^+jj and W^-W^-jj production via weak-boson fusion

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Abstract

We present a next-to-leading order QCD calculation for $e^+\nu_e\mu^+\nu_\mu jj$ and $e^-\bar{\nu}_e\mu^-\bar{\nu}_\mu jj$ production via weak boson fusion at a hadron collider in the form of a fully-flexible parton-level Monte Carlo program, which allows for the calculation of experimentally accessible observables within realistic selection cuts. The QCD corrections to the integrated cross sections are found to be modest, while the shapes of some kinematical distributions change appreciably compared to leading order. The residual scale uncertainties of the next-to-leading order results are at the few-percent level.

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I. INTRODUCTION

Weak-boson fusion (WBF) reactions have been identified as a promising means for gaining a thorough understanding of the electroweak gauge boson sector of the Standard Model (SM) and extensions thereof. WBF is considered as a possible discovery channel for the Higgs boson [1, 2, 3, 4, 5] and a suitable tool for a later determination of its couplings to gauge bosons and fermions [6, 7, 8] and of its CP properties [9, 10]. As WBF reactions are sensitive to the mechanism of electroweak symmetry breaking *per se*, they could help to spot signatures of physics beyond the SM such as strong interactions in the electroweak sector [11, 12, 13, 14, 15, 16, 17, 18]. Furthermore, WBF processes with like-sign dilepton final states are an important background to various new physics scenarios.

In order to unambiguously identify new physics effects, precise predictions for signal and background processes are crucial, including estimates of the theoretical uncertainties. In the context of perturbation theory, the required accuracy and a first assessment of the uncertainties associated with the truncation of the perturbative expansion can only be achieved by the calculation of next-to-leading order (NLO) QCD corrections for experimentally accessible observables. NLO-QCD corrections have therefore been provided for several WBF-induced processes within the SM, including electroweak Hjj , $Hjjj$, $W^\pm jj$, Zjj , W^+W^-jj , $ZZjj$, and $W^\pm Zjj$ production at hadron colliders [19, 20, 21, 22, 23, 24, 25]. QCD corrections have also been determined for selected WBF processes in new physics scenarios, such as [26, 27]. More recently, the electroweak corrections [28] to the Higgs boson signal in WBF have been computed, together with parts of the two-loop QCD corrections [29] and the interference with QCD-induced Hjj production [30, 31]. Supersymmetric corrections to $pp \rightarrow Hjj$ have been presented in the Minimal Supersymmetric Standard Model (MSSM) [32].

In this work, we focus on $e^+\nu_e\mu^+\nu_\mu jj$ production via WBF at a hadron collider and present results for the CERN Large Hadron Collider (LHC). We develop a flexible Monte Carlo program that allows for the calculation of cross sections and kinematical distributions within arbitrary selection cuts at NLO-QCD accuracy. We discuss the most important theoretical uncertainties and quantify the impact of radiative corrections on a few representative distributions.

Results for the $e^-\bar{\nu}_e\mu^-\bar{\nu}_\mu jj$ channel can simply be obtained by charge conjugation and parity reversal. For example, if one is interested in $pp \rightarrow e^-\bar{\nu}_e\mu^-\bar{\nu}_\mu jj$ at the LHC, it is

sufficient to run our code for the process $\bar{p}p \rightarrow e^+\nu_e\mu^+\nu_\mu jj$ with a centre-of-mass energy of 14 TeV, treat the final-state charged leptons as if they were negatively charged, and reverse momentum directions when considering parity-odd distributions. For this reason, in the rest of the paper we will discuss only the $e^+\nu_e\mu^+\nu_\mu jj$ process.

We start with a brief overview of the technical pre-requisites of the calculation in Sec. II. Section III contains our numerical results. We conclude in Sec. IV.

II. TECHNICAL PRE-REQUISITES

The calculation of NLO-QCD corrections to $pp \rightarrow e^+\nu_e\mu^+\nu_\mu jj$ via WBF proceeds along the same lines as our earlier work on WBF W^+W^-jj , $ZZjj$, and $W^\pm Zjj$ production in pp collisions. The methods developed in Refs. [23, 24, 25] can therefore be straightforwardly adapted and only need a brief recollection here.

At order α^6 , WBF $e^+\nu_e\mu^+\nu_\mu$ production in association with two jets proceeds via the scattering of two (anti-)quarks by t -channel exchange of a weak gauge boson with subsequent emission of two W^+ bosons, which in turn decay leptonically. Non-resonant diagrams where leptons are produced via weak interactions in the t -channel also have been included. For brevity, we will refer to $pp \rightarrow e^+\nu_e\mu^+\nu_\mu jj$ as “WBF W^+W^+jj ” production in the following, even though the electroweak production process includes non-resonant diagrams that do not stem from a $W \rightarrow \ell\nu$ decay.

For each partonic sub-process, the 93 contributing Feynman diagrams can be grouped into four topologies, which are depicted for the representative $uc \rightarrow dse^+\nu_e\mu^+\nu_\mu$ mode in Fig. 1. The first two topologies correspond to the emission of two weak bosons from the same (a) or different (b) quark lines with the W^+ decaying leptonically. Topologies (c) and (d) comprise the so-called “leptonic tensors” L_{WW} and $T_{VW,e}$, which describe the tree-level amplitudes of the sub-processes $W^+W^+ \rightarrow e^+\nu_e\mu^+\nu_\mu$ and $VW^+ \rightarrow e^+\nu_e$, respectively. In each case, V stands for a virtual photon or Z boson. Not shown are diagrams analogous to (a) and (d) with W^+ emission from the lower quark line rather than the upper one and graphs which are obtained by permuting the final-state leptons.

We disregard graphs with a weak-boson exchange in the s -channel with subsequent decay into a pair of jets, as they are strongly suppressed in the regions where WBF can be observed experimentally. For sub-processes with identical-flavor combinations such as

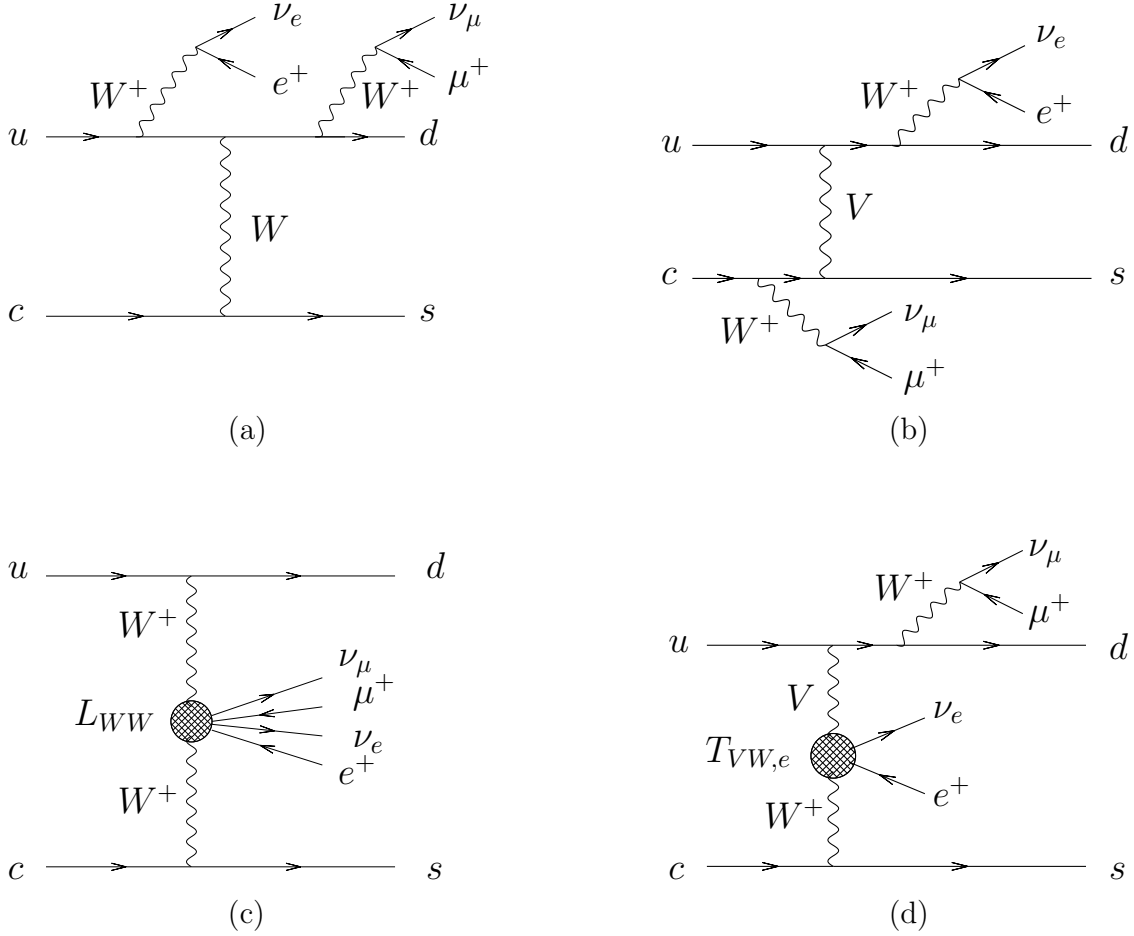


FIG. 1: Feynman-graph topologies contributing to the Born process $uc \rightarrow dse^+\nu_e\mu^+\nu_\mu$. Not shown are diagrams analogous to (a) and (d) with W^+ emission from the lower quark line. V denotes a Z boson or a photon.

$uu \rightarrow dde^+\nu_e\mu^+\nu_\mu$, in addition to the t -channel contributions discussed above also u -channel diagrams and their interference with the t -channel graphs arise. While we do take into account both t - and u -channel contributions, their interference cross section is kinematically strongly suppressed [28] and therefore not considered in the following.

The relevant diagrams are combined in an efficient way, avoiding multiple evaluation of recurring building blocks, and numerically evaluated using the amplitude techniques of Refs. [33, 34].

For the NLO-QCD calculation, real emission and virtual corrections have to be consid-

ered. Infrared singularities are handled via the subtraction formalism in the form proposed by Catani and Seymour [35].

The real-emission contributions are obtained by attaching one extra gluon to the tree-level diagrams sketched above. In addition to (anti-)quark initiated sub-processes such as $uc \rightarrow dsge^+\nu_e\mu^+\nu_\mu$, contributions with a gluon in the initial state (e.g. $ug \rightarrow ds\bar{c}e^+\nu_e\mu^+\nu_\mu$) emerge.

The virtual corrections comprise the interference of one-loop diagrams with the Born amplitude. They are calculated in $d = 4 - 2\epsilon$ dimensions in the dimensional reduction scheme. Due to color conservation, contributions from graphs with the virtual gluon being exchanged between the upper and the lower quark line vanish at order α_s , within our approximations. Only self-energy, triangle, box and pentagon corrections to either the upper or the lower quark line have to be considered. The singularities stemming from infrared divergent configurations are calculated analytically and canceled by the respective poles in the integrated counter-terms of the dipole subtraction approach. The finite pieces of the loop diagrams can be calculated in $d = 4$ dimensions. The emerging two-, three-, and four-point tensor integrals are evaluated numerically by a Passarino-Veltman reduction procedure. In order to avoid numerical instabilities, for the computation of the pentagon integrals we resort to the reduction scheme of Refs. [36, 37], which has already been employed in [25, 38]. We monitor the numerical stability by checking Ward identities at every phase-space point and find that the fraction of events which violate electroweak gauge invariance by more than 10% is at the permille level. The respective phase-space points are disregarded for the calculation of the finite parts of the pentagon contributions and the remaining pentagon part is compensated for this loss. As the pentagon contributions amount to only about 3‰ of the full NLO result, the error induced by this approximation on cross sections and distributions is negligible.

In order to ensure the reliability of our calculation, in addition to the stability tests for the pentagon contributions, we performed the following checks:

- We compared our tree-level and real-emission amplitudes for WBF $e^+\nu_e\mu^+\nu_\mu jj$ scattering and crossing-related processes to the corresponding expressions provided by the automatized matrix element generator MadGraph [39] and found full agreement within the numerical accuracy of our program.
- We compared our integrated cross section for $pp \rightarrow e^+\nu_e\mu^+\nu_\mu jj$ via WBF within the

approximations discussed above to the full cross section provided by MadEvent [39] within typical WBF cuts. Our predictions fully agree with the corresponding MadEvent results within the statistical errors of the two calculations. Agreement between the two programs is also found for representative distributions.

III. NUMERICAL RESULTS

The cross-section contributions discussed above have been implemented in a fully flexible Monte-Carlo program, structured analogous to the **VBFNLO** code [40]. In order to facilitate a comparison of the general features of this production channel, similar settings and selection cuts as in Refs. [23, 24, 25] are employed. We use the CTEQ6M parton distributions [41] with $\alpha_s(m_Z) = 0.118$ at NLO, and the CTEQ6L1 set at LO. Since in our calculation quark masses are neglected, we entirely disregard contributions from external b and t quarks. As electroweak input parameters, we have chosen $m_Z = 91.188$ GeV, $m_W = 80.423$ GeV, and $G_F = 1.166 \times 10^{-5} / \text{GeV}^2$. The other parameters, α_{QED} and $\sin^2 \theta_W$, are computed thereof via tree-level electroweak relations. For the reconstruction of jets from final-state partons we use the k_T algorithm [42, 43, 44, 45] with resolution parameter $D = 0.7$. All our calculations are performed for a pp collider with center-of-mass energy of $\sqrt{s} = 14$ TeV.

In order to clearly separate the WBF signal from various QCD backgrounds, we employ a set of dedicated selection cuts. We require at least two hard jets with

$$p_{Tj} \geq 20 \text{ GeV}, \quad |y_j| \leq 4.5, \quad (1)$$

where p_{Tj} denotes the transverse momentum and y_j the rapidity of a jet j with the latter being reconstructed from massless partons of pseudo-rapidity $|\eta_j| < 5$. The two jets of highest transverse momentum are referred to as “tagging jets”. We impose a large rapidity separation between the two tagging jets,

$$\Delta y_{jj} = |y_{j1} - y_{j2}| > 4, \quad (2)$$

and furthermore demand that they be located in opposite hemispheres of the detector,

$$y_{j1} \times y_{j2} < 0, \quad (3)$$

with an invariant mass

$$M_{jj} > 600 \text{ GeV}. \quad (4)$$

For our phenomenological analysis we focus on the $e^+\nu_e\mu^+\nu_\mu$ leptonic final state. Multiplying our predictions by a factor of two, results for the four-lepton final state with any combination of electrons and/or muons and the associated neutrinos (i.e., $e^+\nu_e\mu^+\nu_\mu$, $e^+\nu_e e^+\nu_e$, and $\mu^+\nu_\mu\mu^+\nu_\mu$) can be obtained, apart from numerically small identical-lepton interference effects.

To ensure that the charged leptons are well observable, we impose the lepton cuts

$$p_{T\ell} \geq 20 \text{ GeV}, \quad |\eta_\ell| \leq 2.5, \quad (5)$$

$$\Delta R_{j\ell} \geq 0.4, \quad \Delta R_{\ell\ell} \geq 0.1, \quad (6)$$

where $\Delta R_{j\ell}$ and $\Delta R_{\ell\ell}$ denote the jet-lepton and lepton-lepton separation in the rapidity-azimuthal angle plane. In addition, the charged leptons are required to fall between the two tagging jets in rapidity,

$$y_{j,min} < \eta_\ell < y_{j,max}. \quad (7)$$

As discussed in some detail in Ref. [25], a proper choice of factorization and renormalization scales, μ_F and μ_R , can help to minimize the impact of higher order corrections on cross sections and distributions in WBF reactions. In the $W^\pm Zjj$ case, the momentum transfer Q between an incoming and an outgoing parton along a fermion line has been found to be more suitable than a constant mass scale. A similar behavior can be observed in the W^+W^+jj case, as illustrated by Fig. 2, where we show the total cross section within the cuts of Eqs. (1)-(7), σ^{cuts} , as function of μ_F and μ_R , which are taken as multiples of the scale parameter μ_0 ,

$$\mu_F = \xi_F \mu_0, \quad \mu_R = \xi_R \mu_0. \quad (8)$$

We distinguish between $\mu_0 = m_W$ and $\mu_0 = Q$ and vary ξ_F and ξ_R in the range 0.1 to 10. At LO, the variation of the cross section with ξ_F resembles the scale dependence of the quark distribution functions, $f_q(x, \mu_F)$, at relatively large values of x . Beyond LO, σ^{cuts} barely depends on μ_F . The renormalization scale enters only at NLO via the strong coupling $\alpha_s(\mu_R)$. Due to the small size of the radiative corrections, the resulting μ_R dependence of the NLO cross sections is marginal. In Tab. I we list the results for σ^{cuts} at LO and NLO for the two scale choices $\mu_0 = m_W$ and $\mu_0 = Q$ with $\xi_R = \xi_F = 1$ and the settings defined above. For reference, we also quote the respective results obtained with the new MSTW08 parton distribution functions of Ref. [46]. It is interesting to note that these results differ

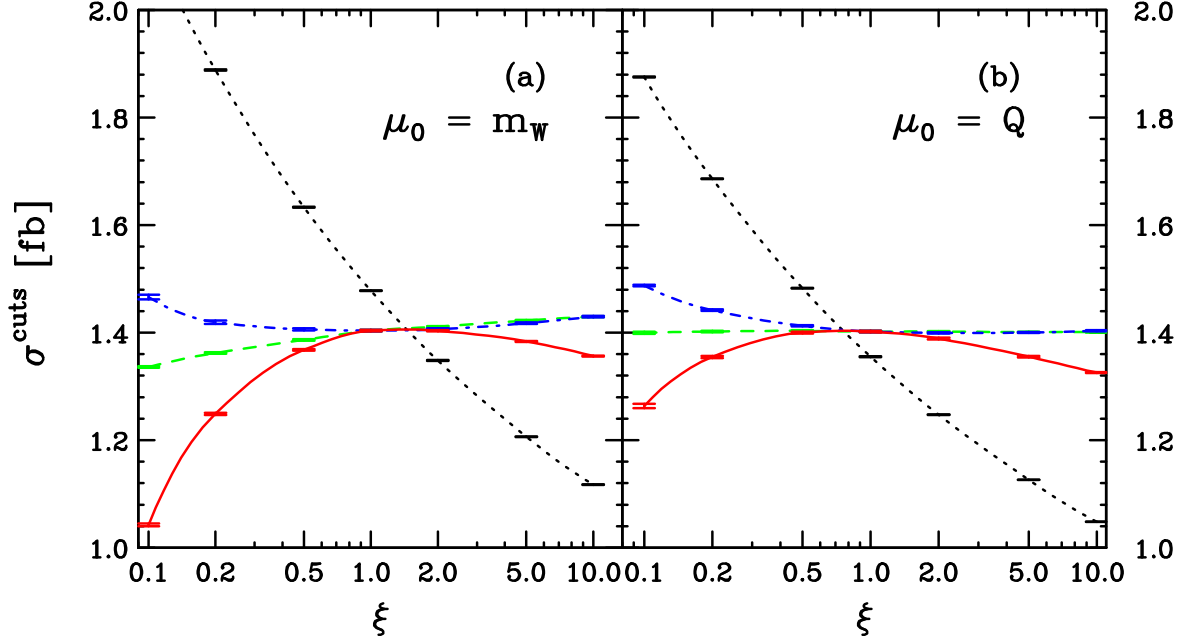


FIG. 2: Dependence of the total WBF $pp \rightarrow e^+\nu_e\mu^+\nu_\mu jj$ cross section at the LHC on the factorization and renormalization scales for the two different choices of μ_0 : $\mu_0 = m_W$ and $\mu_0 = Q$. The NLO curves show σ^{cuts} as a function of the scale parameter ξ for three different cases: $\mu_R = \mu_F = \xi\mu_0$ (solid red), $\mu_F = \xi\mu_0$ and $\mu_R = \mu_0$ (dot-dashed blue), $\mu_R = \xi\mu_0$ and $\mu_F = \mu_0$ (dashed green). The LO cross sections depend only on μ_F (dotted black).

PDF set	$\sigma_{\text{LO}}^{\text{cuts}}(\mu_0 = m_W)$	$\sigma_{\text{NLO}}^{\text{cuts}}(\mu_0 = m_W)$	$\sigma_{\text{LO}}^{\text{cuts}}(\mu_0 = Q)$	$\sigma_{\text{NLO}}^{\text{cuts}}(\mu_0 = Q)$
CTEQ6	1.478 fb	1.404 fb	1.355 fb	1.402 fb
MSTW08	1.409 fb	1.372 fb	1.287 fb	1.369 fb

TABLE I: Cross sections for $pp \rightarrow e^+\nu_e\mu^+\nu_\mu jj$ via WBF at LO and NLO within the cuts of Eqs. (1-7) for the scale choices $\mu_F = \mu_R = m_W$ and $\mu_F = \mu_R = Q$ and two different sets of parton distribution functions. The statistical errors of the quoted results are at the sub-permille level and therefore not given explicitly.

from the corresponding cross sections obtained with the CTEQ6 sets by about 5% at LO and 2 to 3% at NLO. In the following we will stick to $\mu_F = \mu_R = Q$ and the CTEQ6 parton distributions.

A very characteristic feature of WBF-induced processes is the distribution of the tagging jets, which are located in the far-forward and -backward regions of the detector. Due to the

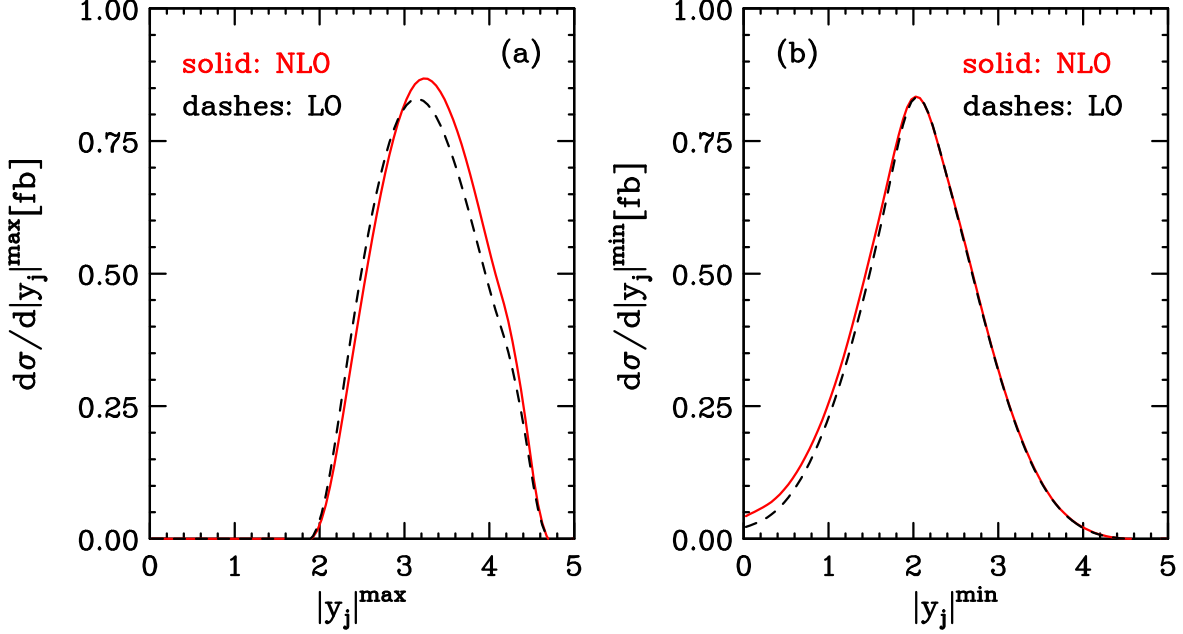


FIG. 3: Rapidity distribution of the jet with the highest (a) and with the lowest (b) value of the absolute value of the rapidity $|y_j|$ in WBF $pp \rightarrow e^+ \nu_e \mu^+ \nu_\mu jj$ production at the LHC at LO (dashed black line) and NLO (solid red line).

color-singlet nature of the t -channel weak boson exchange, the central-rapidity range of the detector exhibits little jet activity. As this feature is an important tool for the suppression of QCD backgrounds which typically exhibit rich hadronic activity at low rapidities [4], it is important to estimate the impact of NLO-QCD corrections on the rapidity distributions of the observed jets. To this end, in Fig. 3 (a) we display the distribution of the jet with the highest absolute value of rapidity at LO and NLO. The respective results for the jet with the lowest absolute value of rapidity are shown in Fig. 3 (b). While at LO each final-state parton corresponds to a tagging jet, at NLO two or three jets may arise due to the extra parton in the real-emission contributions. Because of this extra parton, the probability to find a jet of very high or very low rapidity is larger at NLO than at LO.

In Fig. 4, the invariant mass distribution of the two tagging jets is shown for two choices of μ_0 together with the dynamical K factor, which is defined via

$$K(M_{jj}) = \frac{d\sigma_{\text{NLO}}/dM_{jj}}{d\sigma_{\text{LO}}/dM_{jj}}. \quad (9)$$

The M_{jj} distribution was scrutinized in Ref. [25] to demonstrate the impact of the choice of factorization and renormalization scales on the perturbative stability of calculations for

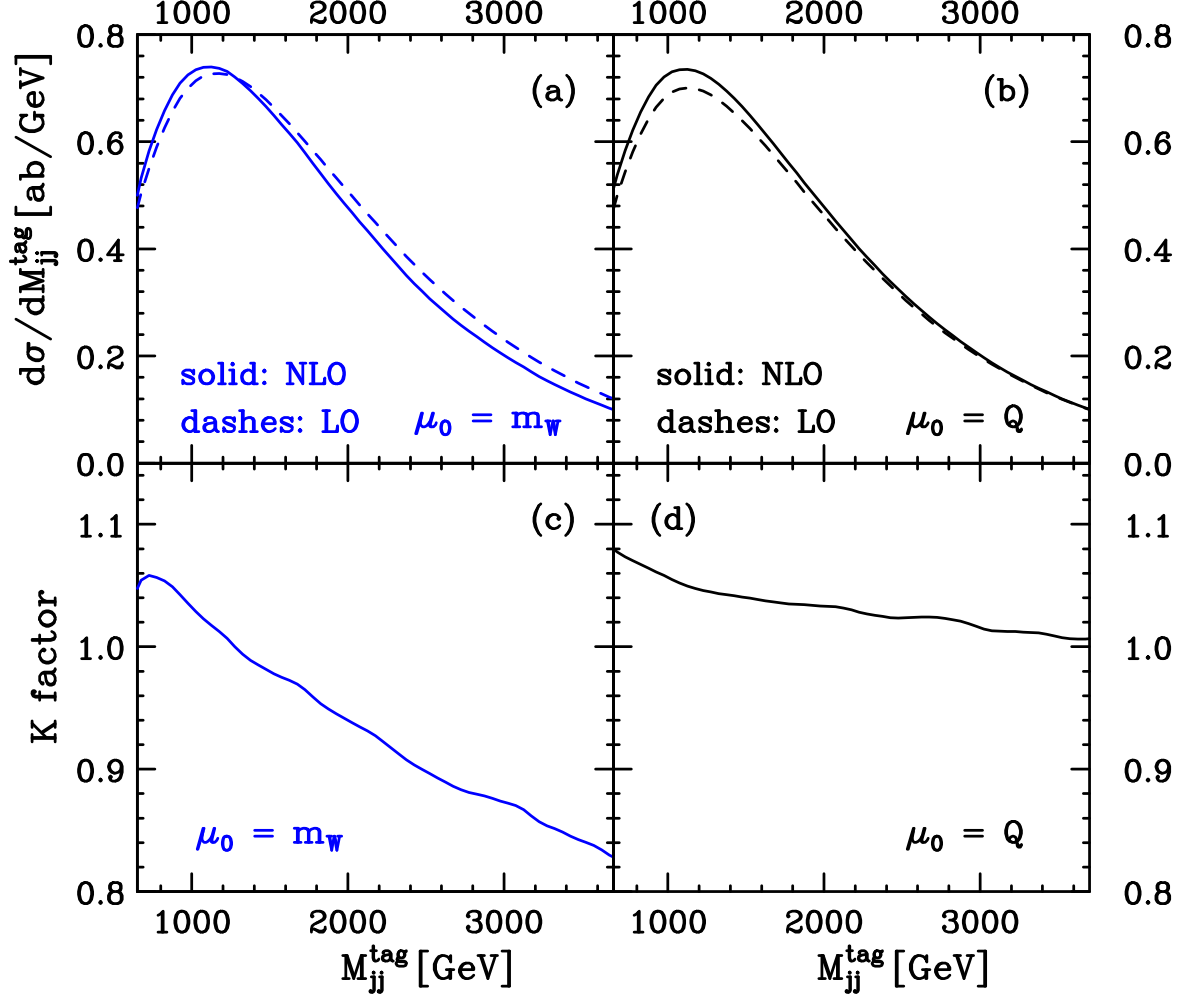


FIG. 4: Invariant mass distribution of the tagging jets in WBF $pp \rightarrow e^+\nu_e\mu^+\nu_\mu jj$ production at the LHC for two different choices of μ_0 [panels (a) and (b)] at LO (dashed) and NLO (solid). Their ratios, the K factors as defined in Eq. (9), are displayed for $\mu_0 = m_W$ in panel (c) and for $\mu_0 = Q$ in panel (d).

WBF processes. Using the example of WBF W^+Zjj production it was shown that small NLO-QCD corrections to $d\sigma/dM_{jj}$ are obtained, if μ_F and μ_R are identified with Q . A similar behavior is observed for the W^+W^+jj case, with a K factor close to one over the entire range of M_{jj} for $\mu_0 = Q$, but pronounced shape distortions at LO for $\mu_0 = m_W$.

IV. SUMMARY AND CONCLUSIONS

In this work, we have presented a NLO-QCD calculation for WBF W^+W^+jj and W^-W^-jj production at the LHC, which takes leptonic decays of the weak gauge bosons fully into account. We have developed a flexible Monte-Carlo program that allows for the calculation of cross sections and distributions within typical WBF cuts. The QCD corrections to the integrated cross section are modest, changing the leading-order results by less than about 7%. The residual scale uncertainties of the NLO predictions are at the 2.5% level, indicating that the perturbative calculation is under excellent control. In accordance with our earlier work on WBF $W^\pm Zjj$ production we found that size and shape of the NLO results can be best approximated by the corresponding LO expressions, if the momentum transfer, Q , of the t -channel electroweak boson is chosen as factorization scale.

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